LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2013**

ST 2502/ST 2501 - STATISTICAL MATHEMATICS - I

Date: 30/04/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

PART – A

Answer **ALL** the questions: (10 x 2 = 20)

1. Define Convergent Sequence.
2. Define a bounded sequence
3. Define absolute convergence of a series.
4. Define partial sum of a series.
5. Examine if Rolle's Theorem is applicable to the function f(x)=x, $0 \le x < 1$, and f(1)=0.
6. Define probability generating function.
7. Define linearly dependent set of vectors.
8. State the properties of probability mass function
9. Define rank of a matrix
10. Define orthogonal matrix.

PART - B

Answer any **FIVE** questions: (5 x 8 = 40)

11. Show that $\lim_{n\to\infty} \frac{n^2+1}{2n^2+3} = \frac{1}{2}$.
12. Define cumulative distribution function and write down its properties.
13. Show that the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}...$ is convergent and its sum is 1.
14. Discuss the convergence / divergence of (i) $\sum_{n-1}^{\infty}\frac{1}{n}$ (ii) $\sum_{n-1}^{\infty}\frac{1}{n^2}$.
15. Show that $M_{X1+X2+..+Xn}(t) = M_{X1}(t).M_{X2}(t)...M_{Xn}(t)$, where $X_1, X_2, ..., Xn$ are independent random variables and M denotes moment generating function.
16. Verify whether (1,2,4,), (-1,2,0), (-1,6,4) are linearly dependent or independent
17. Verify whether the Matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.
18. Find the inverse of the martin A = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

PART - C

Answer any **TWO** questions: (2 x 20 = 40)

19. i) Show that every convergent sequence is bounded. Is the converse true? Justify your answer. (10 marks)

ii) Let X be a continues random variable with p.d.f. given by $f(x)\begin{cases} kx, & 0 \le x < 1 \\ k, & 1 \le x < 2 \\ -kx + 3k, & 2 \le x < 3 \\ 0, & otherwise \end{cases}$

Determine the constant k and the c.d.f. F(x). (10 marks)

20. i) State and prove Rolle's Theorem.

ii) Find the first four moments of a distribution whose m.g.f.is $M_X(t)=\exp\{4(e^t - 1)\}$.

21. i) If $f(x , y)=\dfrac{x+y}{x^2+y^2}$ find $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial^2 f}{\partial y^2}$.

ii) The joint probability distribution of two random variables X and Y is given by

$P[X=0, Y=1]=\dfrac{1}{3}$, $P[X=1, Y=-1]= \dfrac{1}{3}$ and $P[X=1, Y=1]= \dfrac{1}{3}$.

Find a) Marginal distributions of X and Y.

b) Conditional distribution of X given Y=1.

22. i) Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$.

ii) Two random variables X and Y have the following joint p.d.f.

$f(x, y)=\begin{cases} 2-x-y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & otherwise \end{cases}$

Find Var (X), Var (Y) and Cov (X,Y).

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